Jordan type maps, symmetries, and (piecewise) structures of operator algebras

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Jordan type map between algebraic structures is a map φ preserving the squares ($\varphi(a^2) = \varphi(a)^2$). Long history of theory of matrix and operator algebras has shown that Jordan maps embody symmetries of different structures connected with operators (metric structure, state spaces, projection lattices, order unit structures, etc.). This universality of Jordan isomorphisms is surprising. We would like to illustrate this fact by presenting new recent results along this line. In particular, we shall consider piecewise structures associated with C*-algebras. Roughly speaking, piecewise structure results from general algebraic structure by forgetting a given operation for non-commuting elements. We show that the system $C(A)$ of commutative C*-subalgebras of a C*-algebra A, ordered by set theoretic inclusion, determines (as a mere poset) piecewise Jordan structure of A. With the help of Generalized Gleason Theorem we show that for typical von Neumann algebras and AW*-algebras poset isomorphisms between $C(A)$'s are implemented by linear Jordan isomorphisms. Related results on further operator theoretic structures, including decompositions of states and Choquet order, are discussed. Next we show that Jordan isomorphisms are important for describing non-linear preservers for various (non-equivalent) orders on matrices and operators. Besides, connection of the presented results with problems in interpretation of quantum mechanics (Bohrification program) is outlined. The exposition should be intuitive and non-technical.

References

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